The Development of an Assessment Tool: Student Knowledge of the Concept of Place Value

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The importance of student understanding of the concept of place value cannot be underestimated. Place value is a 'gate keeper' in developing mathematical understanding. The purpose of this study was to examine and develop a teacher-made test of place value knowledge. The questions were developed using the progressions from the Number Framework (Bobis, Clarke, Clarke, Thomas, Wright et al., 2005). An exploratory factor analysis was used to evaluate the assessment tool. The analysis of student responses to the test questions revealed a three-factor structure that supported the existing literature on the progression in learning of place value ideas, by identifying the critical key ideas that underlie the concept of place value. The results validated the tool as a test of place value knowledge that could be used to assess the performance of Yrs. 3-9 students.

Introduction

In 2005 a cluster of seven schools formed a professional learning community (PLC) with a common goal: to raise the mathematical achievement of all students within their school community. The schools' numeracy lead teachers began to meet regularly for in-depth discussions on reaching this goal. All these schools had previously participated in a national professional development programme in mathematics (Ministry of Education, 2007). Teacher awareness of the 'gaps' in students' mathematical knowledge began to emerge through participation in this project. Student knowledge of place value was perceived as a barrier to further learning in mathematics. The PLC decided, therefore, to focus on place value. As a first step they wanted to know more about what the students knew about place value, across the seven schools. To this end they asked the author to develop a tool that would provide information about the students in Years 3 to 9 thereby providing a suitable starting point for teaching and learning programmes. This paper reports on the performance and development of the assessment tool that was designed to meet this community's needs (Major, 2011).

Background

Place value is a necessary and fundamental mathematical concept for student success in mathematics and as such is a gatekeeper to further mathematical understanding. The complexity of place value understanding is masked by the simplicity of its modern formulation, which condenses multiple key ideas into an underlying construct. Simply defined as a way of naming or representing numbers, place value provides a structure that allows us to write and say numbers, allowing for both whole numbers and parts of numbers to be represented, and makes mental computation possible. The significance of place value knowledge is evident in our ability to read, write and understand large numbers, carry out complex computations and express the results of our calculations in a variety of forms. Place value is conceptually embedded in any mathematical operation students need to learn, and it is this embedding that makes the teaching and learning of this concept challenging. Research evidence suggests that students can appear to have mastered aspects of place value yet have no real understanding of the concept, being unable to generalise the multiplicative relationships within the place value system (Irwin, 1996; Kamii, 1986; Thomas, 2004).

In J. Dindyal, L. P. Cheng & S. F. Ng (Eds.), Mathematics education: Expanding horizons (Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia). Singapore: MERGA.

Research has identified relevant knowledge that is necessary for the development of an understanding of place value. Student knowledge of counting patterns and strategies is markedly related to the knowledge of, and ability to explain, place value (Boulton-Lewis, 1996: Rubin & Russell, 1992; Young-Loveridge, 2008). Young-Loveridge (1998) argued that children can be taught place value when they understand the concept of ones and have constructed number relationships that will support the concept of ten as a unit. Thomas and Ward (2002) found that older children introduced to place value concepts made greater gains than their younger counterparts; perhaps suggesting, like Kamii (1986), that there is a link between age and place value conceptual development. Student progression in the construction of knowledge about place value and the number system has been described as often unpredictable and non-linear (Thomas, 1996). Most of the literature appears to agree that place value concepts should be taught gradually through children developing mental strategies for solving multi-digit addition and subtraction problems (Beishuizen & Anghileri, 1998; Cobb & Wheatley, 1988; Fuson, Wearne, Hiebert, Murray, Human et al., (1997); Kamii, 1986; Thompson, 2000).

A number of key constructs have been identified to describe place value understanding for teaching purposes (Fuson et al., 1997; Jones, Thornton, Putt, Hill, Mogill et al., 1996; Wright & Gould, 2000; Young-Loveridge, 1999). Although these constructs differ in the number of developmental stages they define they do have some common principles that highlight the complexities of the interrelationships between counting, knowledge of number, grouping and partitioning. The most common principle is that children progress through a number of well defined stages as their thinking develops from early counting-in-ones knowledge to a more sophisticated understanding of multi-digit numbers and their use in problem solving. The time children spend using strategies to work out problems facilitates the learning of knowledge, that when mastered becomes automatic. A student may solve the problem 10+4 many times by ‘holding’ 10 in their head and counting on four more before the place value understanding of the two parts, consisting of one ten and four ones making one whole number of fourteen becomes an automatic response.

Teacher knowledge of mathematics is an essential component of effective teaching (Anthony & Walshaw, 2007; Ball, Hill & Bass, 2005; Thomas & Ward, 2007; Young-Loveridge & Mills, 2009). Effective teaching of place value requires an understanding of the learning progressions and how to assess against these progressions. The Number Knowledge Framework (Ministry of Education, 2007, p.18) highlights place value by providing a basic progression of place value knowledge detailed in terms of ‘grouping’ ideas. However, the progression does not include details of the key principles of place value. In fact the apparent clarity and simplicity of the framework belies the complex understandings implied by the learning outcomes.

Assessments tools currently available to New Zealand’s teachers do not exclusively test place value knowledge, although many number test items include place value ideas. The literature on the learning and teaching of place value is clear about its importance for learners. Teachers need good information about their students’ understanding of place value in order to improve their instructional practices and student outcomes. For this mathematics PLC there was a need for a common assessment tool that was easy to administer, and would give teachers good information about their students understandings from which they could plan for student learning. The purpose of this study, therefore, was to find out how effectively students’ knowledge of place value could be assessed by a timed, short answer, written test.

Method

One thousand and forty one students were selected from seven schools, three primary (Yrs 0-6), two full primary (Yrs 0-8), one intermediate (Yrs 7-8) and one college (Yr 9). Six of these schools are in the same geographic area, are low-decile and were members of the Mathematics Cluster. To increase the number of test results available for the factor analysis, one mid-decile primary school that was not a member of the Cluster or in the same geographic area, was invited to participate. Students selected to participate were those attending school on the day of the testing. These participants ranged in age from eight to fourteen years of age. The sample consisted of 50% boys and 50% girls. Ethnic composition of participants by year levels, are shown in Table 1.

Table 1
Ethnicity of Participants by Year Levels

Year Levels	*Maori*	*NZ European*	*Pacific Islands*	*Asian*	*Other*
3 - 6	474	257	14	14	18
7 - 8	163	11	4	0	3
9	78	2	2	0	1

A place value test consisting of five sections was devised by the author to assess students' knowledge of place value concepts, based on stages four to eight of the Number Knowledge Framework (Ministry of Education, 2007). Each of the five sections included ten questions derived from that stage. For example, the first section contained questions pertinent to stage four of the Number Knowledge Framework. The items were designed to test the students' early knowledge of place value through groupings within ten and twenty and the number of tens in decades, which required knowledge of counting in ones over a decade, and grouping in and with ten using words and symbols. The design of the items in the following two sections (Stages 5 and 6) extended on this knowledge to groupings within 1000, groupings of tens and hundreds within four digit numbers and rounding whole numbers to the nearest ten, hundred or thousand. These sections also contained several decimal items designed to test students' knowledge of the number of tenths and hundredths in decimals to two places and rounding to the nearest whole number, of decimals with up to two places. The final two sections (Stages 7 and 8) tested students' place value knowledge, again building on the knowledge required for the preceding sections, of decimal fractions. Items were written to reflect the aspects of place value highlighted by the Number Knowledge Framework (Ministry of Education, 2007).

The test was administered to classes via a timed voice-over power point. At face value this test appears to be purely a test of knowledge, as the test is timed and requires students to instantly recall their knowledge. However, students who understand the concepts, rather than having superficial techniques for item types, will be more successful in the test. Students using their knowledge of place value concepts to solve items are more likely to solve the item within the allocated time, than students who try to use a learned procedure (procedural knowledge) to work it out. For example, a student who understands that 8 tens is the same as eighty and that twenty is the same as 2 tens is likely to solve the problem 8 tens + 20 = more quickly than a student who uses skip counting knowledge 10, 20, 30,

….90, 100. Students who understand the concept of ten as a unit are more likely to give a rapid response.

Students recorded their answers to the test on a tabulated recording sheet. The student responses, marked correct or incorrect, were converted into numerical code for the purpose of a statistical analysis using SPSS software, (SPSS Inc., 1998). To determine whether a factor analysis was appropriate, the assumptions that underlie the procedure were tested. Firstly an alpha coefficient was calculated to test for internal consistency. This ensured that the items were sufficiently related for further exploration of underlying factors to be worthwhile. Alpha was equal to 0.95, indicating that further analysis was warranted. The KMO test of sampling adequacy yielded a value of 0.915. This result indicates that sampling parameters were sufficient for a factor analysis to be conducted. There were 1041 responses to the test, yielding subject to item ratio of greater than 20:1. This means that there were sufficient responses to each item for the results of a factor analysis to be useful. An exploratory factor analysis was undertaken because it was not known how many factors might underlie students' performance on the test. The structure of the Number Framework suggested that all the item types were measures of place value understanding, but how these might be clustered in students' performance was unknown. The exploratory factor analysis enabled the author to find a best-fit solution for the data set.

The data set was non-normal, and entered as category data (1 for a correct answer and 0 for incorrect). For this reason a principal axis factoring (PAF) extraction method was used (Costello & Osbourne, 2005). The scree plot of Eigenvalues was used to establish the number of factors for rotation. The point at which the scree slope flattened suggested that there were three factors present in the data set.

After extraction, a direct oblimin rotation was performed. An oblimin rotation is an oblique rotation that allows correlation between the resulting factors. This was appropriate because the test was constructed to test an underlying concept, and any factors would be expected to correlate. The resulting factor loadings are presented in Table 2.

Factor loadings of 0.32 or greater are commonly regarded as adequate for establishing the existence of a factor (Tabachnick & Fidell, 2001). Items that load greater than 0.32 on more than one factor are cross-loaded. Items strongly loaded on two factors cannot be regarded as distinct. Cross-loaded items are identified in Table 2.

Table 2

Factor Loadings >0.32 Based on a Principal Axis Factoring (PAF) Factor Analysis with an Oblimin Rotation of 50 Items from the Place Value Test (N=1041)

Item	*Factor 1*	*Factor 2*	*Factor 3*
1. 7+10	0.614		
2. 10+?=18	0.664		
3. Write 14 as a number	0.529		
4. 20=13+?	0.371	0.486	
5. 8 tens is the same as what number?	0.629		
6. How many 10s in 38?	0.487		
7. How many ones in 65?	0.335		
8. 5 tens and 2 ones = ?	0.586		
9. 6 tens and 30 = ?	0.463	.409	
10. What number comes next 43,42,41, ?	0.538		
11. Write the number one hundred and forty three	0.692		
12. 604= 4 +?		0.562	
13. How many tens in all of this number? 836		0.507	
14. 50+300+? = 354	0.379	0.494	
15. Round to nearest 10. 598		0.596	
16. How many hundreds in all of this number? 5000		0.477	
17. 20+300+8000+6=		0.580	
18. 697+4= ?		0.580	
19. 596 – 100 =		0.630	
20. 3508 -10=		0.503	
21. How many tens altogether in this number? 3607		0.530	
22. Round to nearest 100. 8574		0.741	
23. How many tenths altogether in four point two three?			0.456
24. How many hundredths in all of thirty point zero six?			0.733
25. Round this number to the nearest whole number. 40.98		0.592	
26. Write the missing number: 830+?=1000		0.633	
27. 0.70 is the same as 7 tens, 7 ones or 7 tenths?		0.547	
28. 4000/10		0.608	
29. 1011 x 8		0.632	
30. 32 x 10		0.669	
31a. Smallest number of 762,123; 89,549; 79,532; 817,300		0.629	
31b. Largest number of 762,123; 89,549; 79,532; 817,300		0.741	
32. Round this number to the nearest tenth – 8.79		0.631	
33. Round this number to nearest 100th – 5.468			0.543
34. How many thousands in all of this number: 814 342		0.590	
35. 0.5+0.001+0.09=			0.469
36. 3.2x 100			0.624
37. 64.2/10			0.707
38. 6.813 + 1/10			0.564
39. Write a number between 0.1 and 0.2		0.347	0.407
40. Which decimal fraction has the greatest value? 0.3, 0.299		0.585	
41. Round this number to the nearest hundred – 3254		0.623	
42. Round this number to the nearest hundredth – 9.106			0.320
43. How many thousandths in all of this number? 4.182			0.692
44. Write these decimal fractions in order, smallest first: 0.29, 0.6, 0.371		0.481	
45. 7.04-0.1=			0.478
46. 0.29 x 10			0.674
47. 48/100			0.761
48. 0.18/10			0.775
49. 17.9 + 1/100			0.652
50. 77 divided by 10			0.518

Results and Discussion

The test was constructed to try to assess a single, underlying concept. The exploratory factor analysis, however, revealed three underlying factors. The pattern that emerged was in line with the literature on place value. An outline of these factors is presented in Figure 1.

Factor 1	Ones and ten-structured concepts using words, digits and symbols – counting by ones and grouping in and with tens
Factor 2	Multi-unit concept – grouping and regrouping of, and within, 3+ digit whole numbers, and rounding to whole number or nearest tenth (2 decimal places)
Factor 3	Extended multi-unit concept – whole numbers and decimals and the relationship between whole numbers and decimals, and simple fractions.

Figure 1. Labels and descriptions of the three identified factors

Factor one is made up of nine items (1-3, 5-8, 10-11) that link to unitary and ten-structure concepts, for example, counting back by one to find the number before, and knowing that two digit numbers are made up of groups of tens and ones. These items include knowledge of the relationship between the number words and digits and the language of operations and their corresponding symbols. The twenty-one questions in factor two (12-13, 15-22, 25-32, 34, 41) are consistent with a multi-unit concept. These questions required students to group and/or regroup whole numbers using part whole concepts and their conceptual knowledge of the base-ten principle – the knowledge that the values of the positions increase or decrease in powers of ten. Factor three (items 23-24, 33, 35-38, 42-43, 45-50) is an extension of the multi-unit concepts involving the relationship between whole numbers and decimals, and their relationship with simple fractions. The analysis identified four items (4, 9, 14, 39) that were cross-loaded on two factors above the factor threshold of 0.32. These items were removed from the test. A further two items (40, 44) were removed as they were questions with multiple parts. Selected response questions or those where subsequent answers were dependent on earlier answers were deleted as they did not differentiate between those students who knew the answers and those that guessed, making these items unreliable.

It was initially assumed that the test items would assess a single underlying concept and therefore items would be grouped onto one factor. The factor analysis showed that most items were indeed closely related. The strong loading of items onto three factors rather than one implies that place value is as complex as the literature suggests.

One factor is made up of whole number items that assess the concepts of counting, and early knowledge of grouping in ones and tens and how these numbers are written. Factor two extends this knowledge into numbers within one thousand and an early knowledge of decimal place value. The third factor contains decimal fraction and fraction items that require a conceptual knowledge of the multiplicative principle of place value. The makeup of these factors is not necessarily surprising. The literature argues that children need a good understanding of whole number place value concepts before moving on to working with decimal numbers (Irwin, 2001; Sowder, 1997; Wearne & Kouba, 2000). The factor analysis supported the literature that children progress through a number of increasingly complex stages as their thinking develops, and identified a structure of three factors through which to measure this development. The structure contained similar principles to those identified by the literature (Fuson et al, 1997; Jones et al, 1996; Wright & Gould, 2000; Young-Loveridge, 1999).

In terms of alignment with the Number Framework (Ministry of Education, 2007) items in factor one match stage four with the exception of one item (11 stage five). Factor two consists of stage five and six items with the addition of one item (41) from stage 7. Most of factor three items are consistent with stages seven and eight, items 23 and 24 were developed from stage six. The test items were presented in the test in Number Framework stages to illustrate the progression of place value knowledge and so that students correct responses within the stages would give an indication of which stage the student was most likely to be working within. The factor analysis, in clustering the items into the three factors, meant that students could no longer be assigned a single stage for place value against the Number Framework. It would appear to be more useful for teachers to think about next steps for teaching and learning in terms of progress through the ones and ten-structured concept, multi-digit concept and extend multi-digit concept, as presented within the three factors.

Conclusion

The analysis of the place value test used in this study showed that the items did in fact reliably assess the concept of place value. The factor information provided a breakdown of the critical key ideas underlying the concept of place value and supported the existing literature (Fuson et al., 1997; Jones et al., 1996; Wright & Gould, 2000; Young-Loveridge, 1999). The breakdown was consistent with the Number Framework in terms of grouping the stages, for example, factor two reflects a multi-digit concept consistent with stages five and six (Major, 2011). These three factors further specify the progression of understanding suggested by the literature and the Number Framework, and could therefore inform the teaching of place value. Such teaching would approach the concept of place value through the underlying critical key ideas, supporting the development of a robust understanding in students that creates a platform for their further mathematical understanding. This action has the potential to raise student achievement in mathematics – the goal of the Mathematics Cluster.

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